

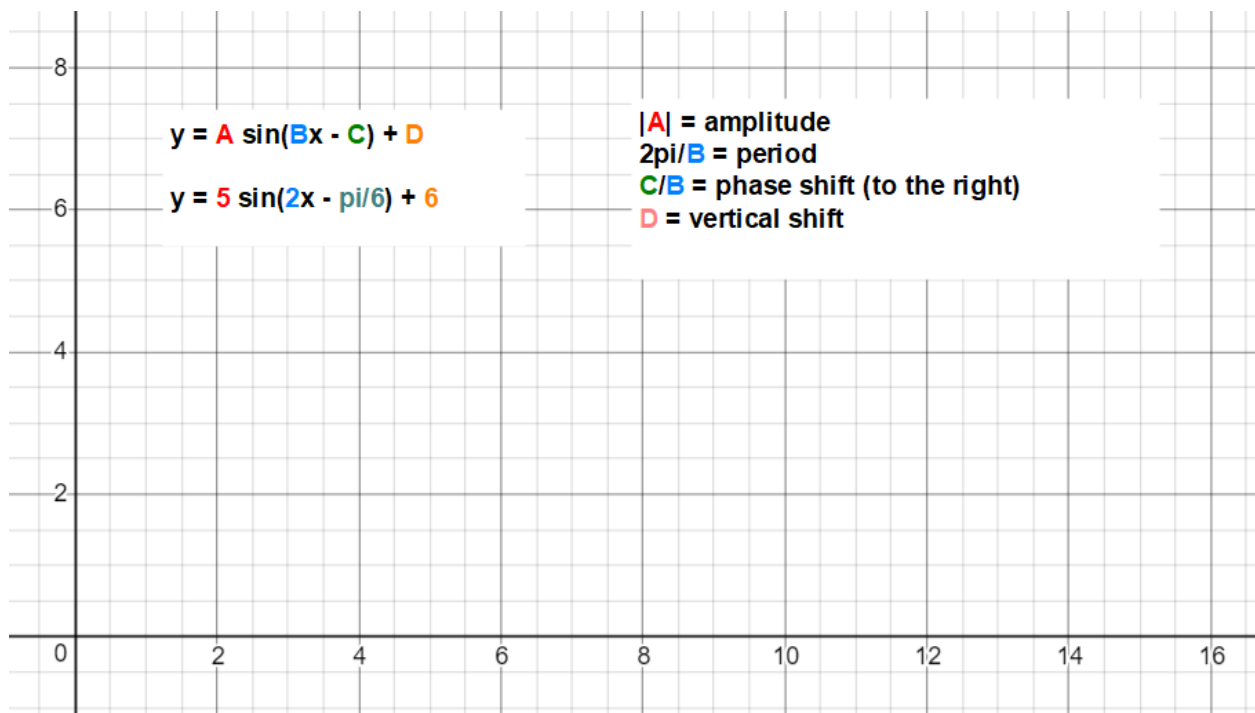
## Simple Periodic Motion – Amplitude, Period, Phase Shift, Vertical Shift

The first function below is the general form of a periodic function what shows the **A** (amplitude), **B** (the constant used to calculate period), **C** (the constant used to calculate the phase shift), and **D** (vertical shift).

$$y = A \sin(Bx - C) + D$$

The second function below is a specific example with specific constants for **A**, **B**, **C**, and **D**:

$$y = 5 \sin(2x - \pi/6) + 6$$

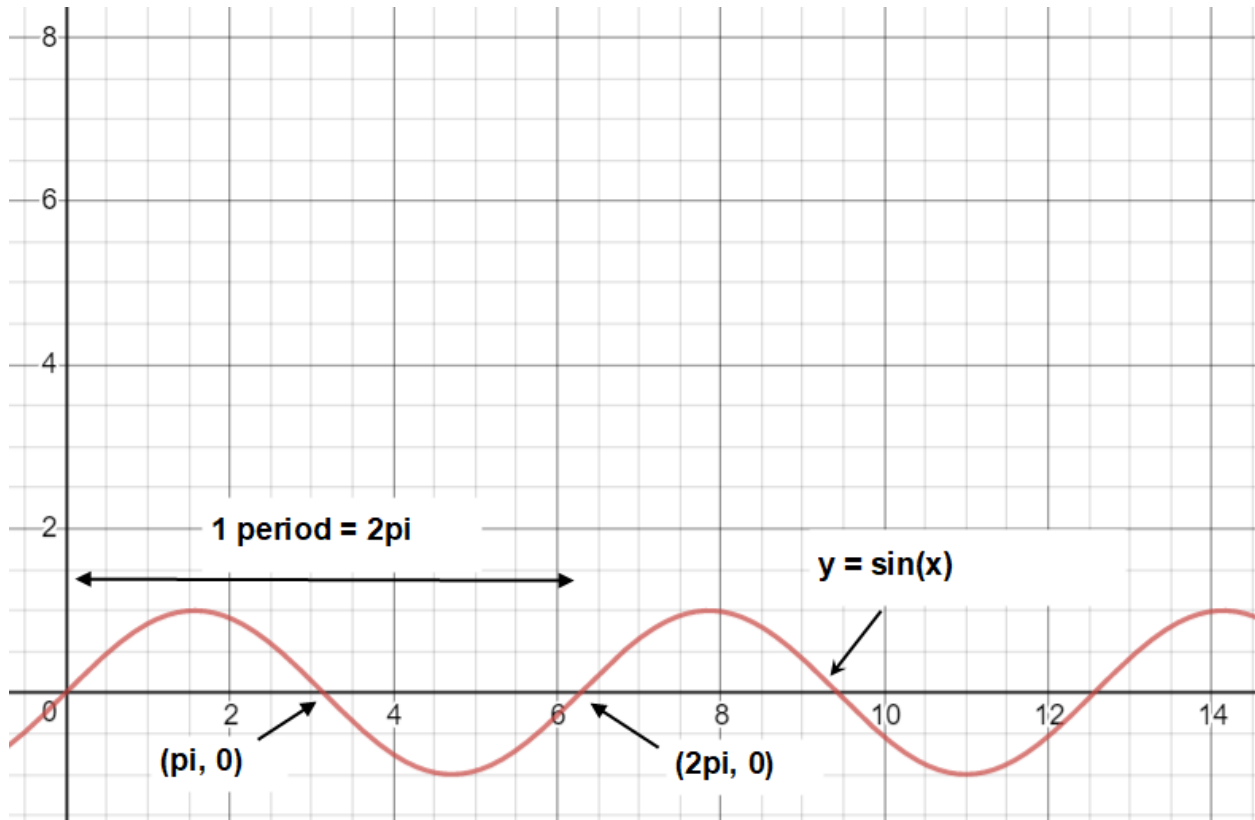


On the following pages, each constant (**A**, **B**, **C**, **D**) will be used, one at a time, to change the simple function:  $y = \sin(x)$  incrementally.

## Graphing ... one step at a time ... start with the parent function

**Step 1:** Graph the parent function (the simplest form of the sine function):

$$y = \sin(x)$$

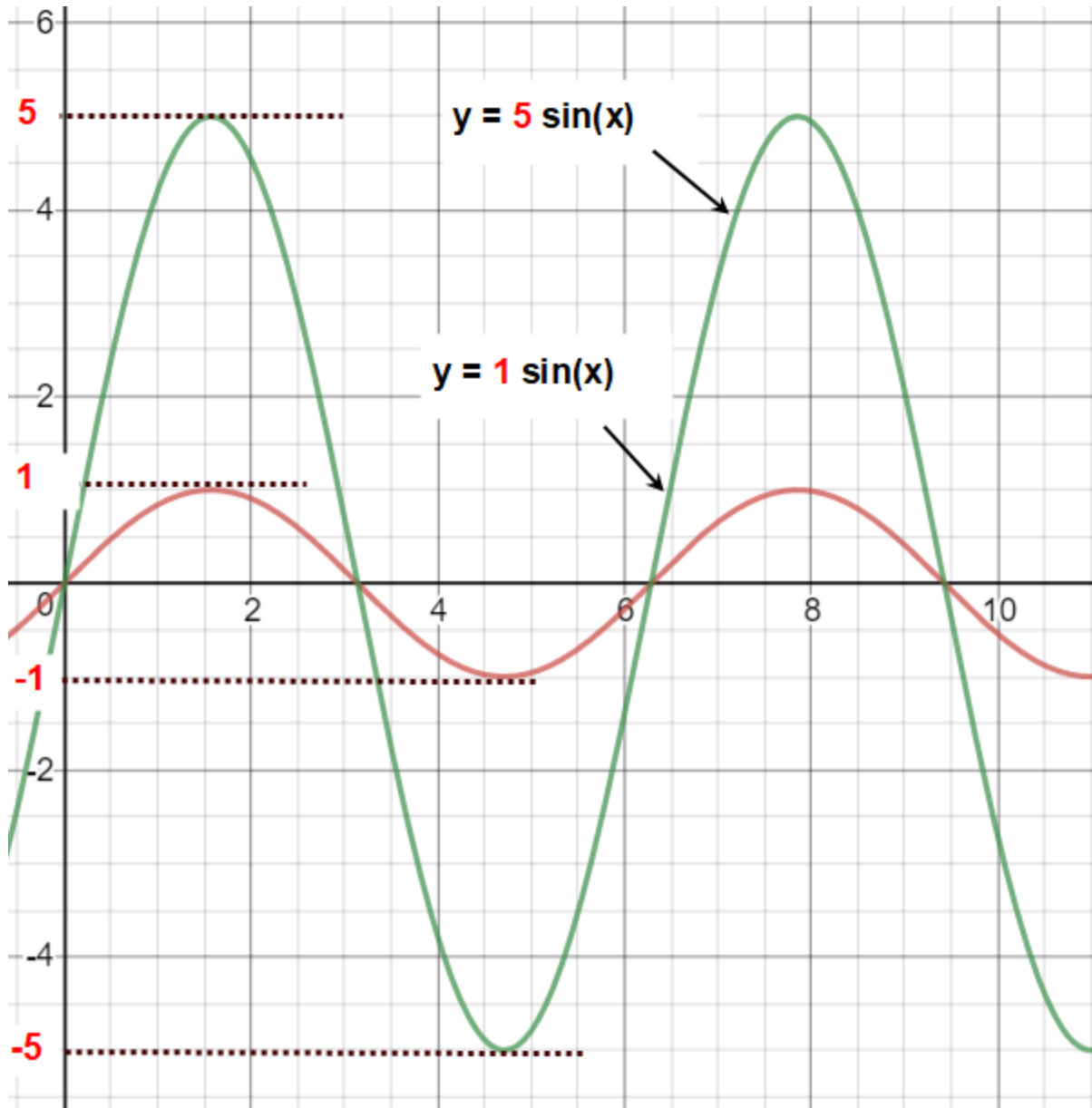


Note that the period (how many radians before the cycle repeats) is  $2\pi$  for this parent function.

## Graphing ... one step at a time ... applying the new amplitude

Step 2: Change the amplitude from 1 to 5 resulting in:

$$y = 5 \sin(x)$$

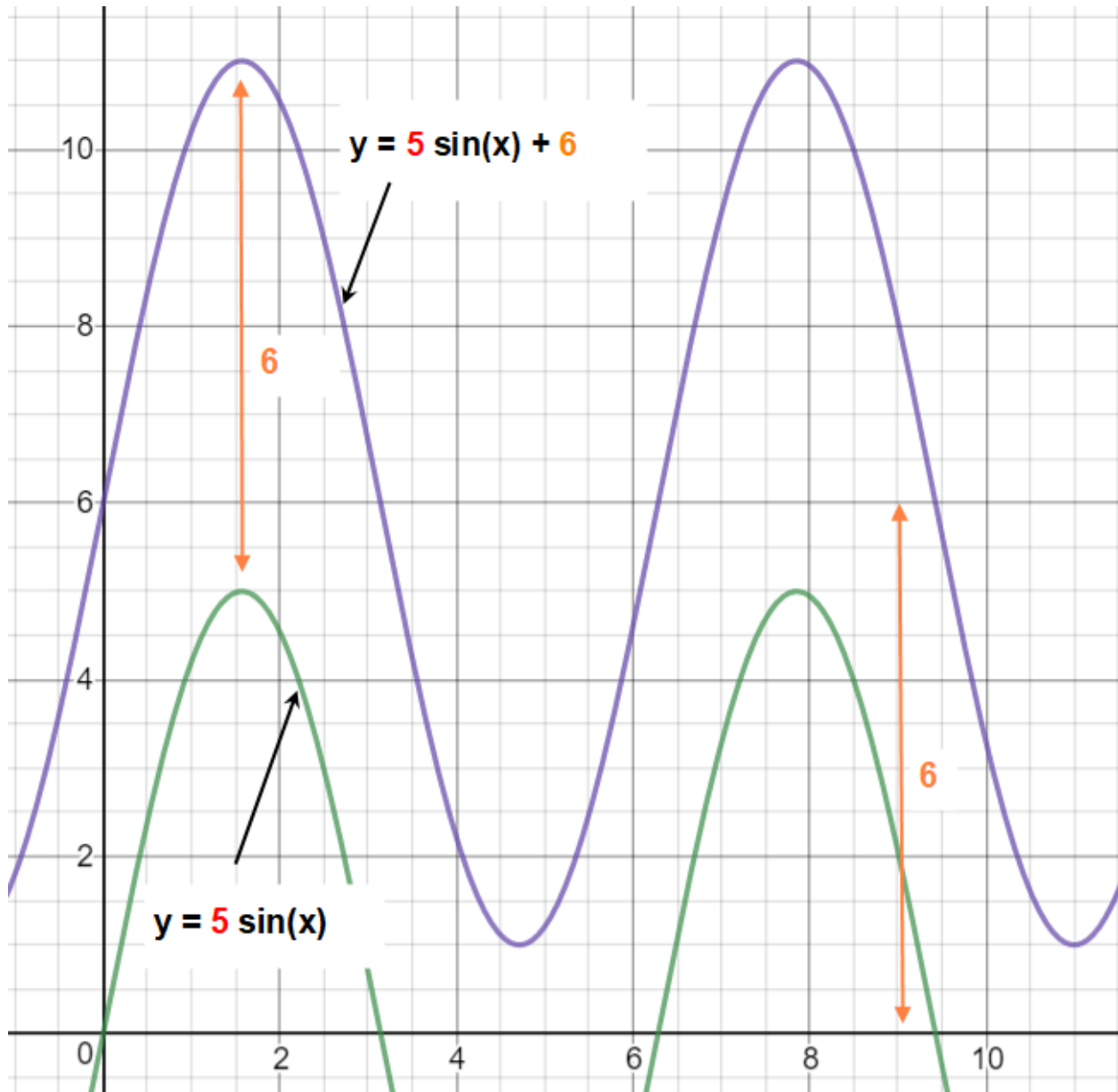


Above, the amplitude of 1 for the parent function ( $y = \sin(x)$ ) becomes 5. Note that the period has not changed, only the amplitude.

## Graphing ... one step at a time ... applying the new vertical shift

**Step 3:** Shift the entire graph of  $y = 5 \sin(x)$  up 6 units resulting in:

$$Y = 5 \sin(x) + 6$$

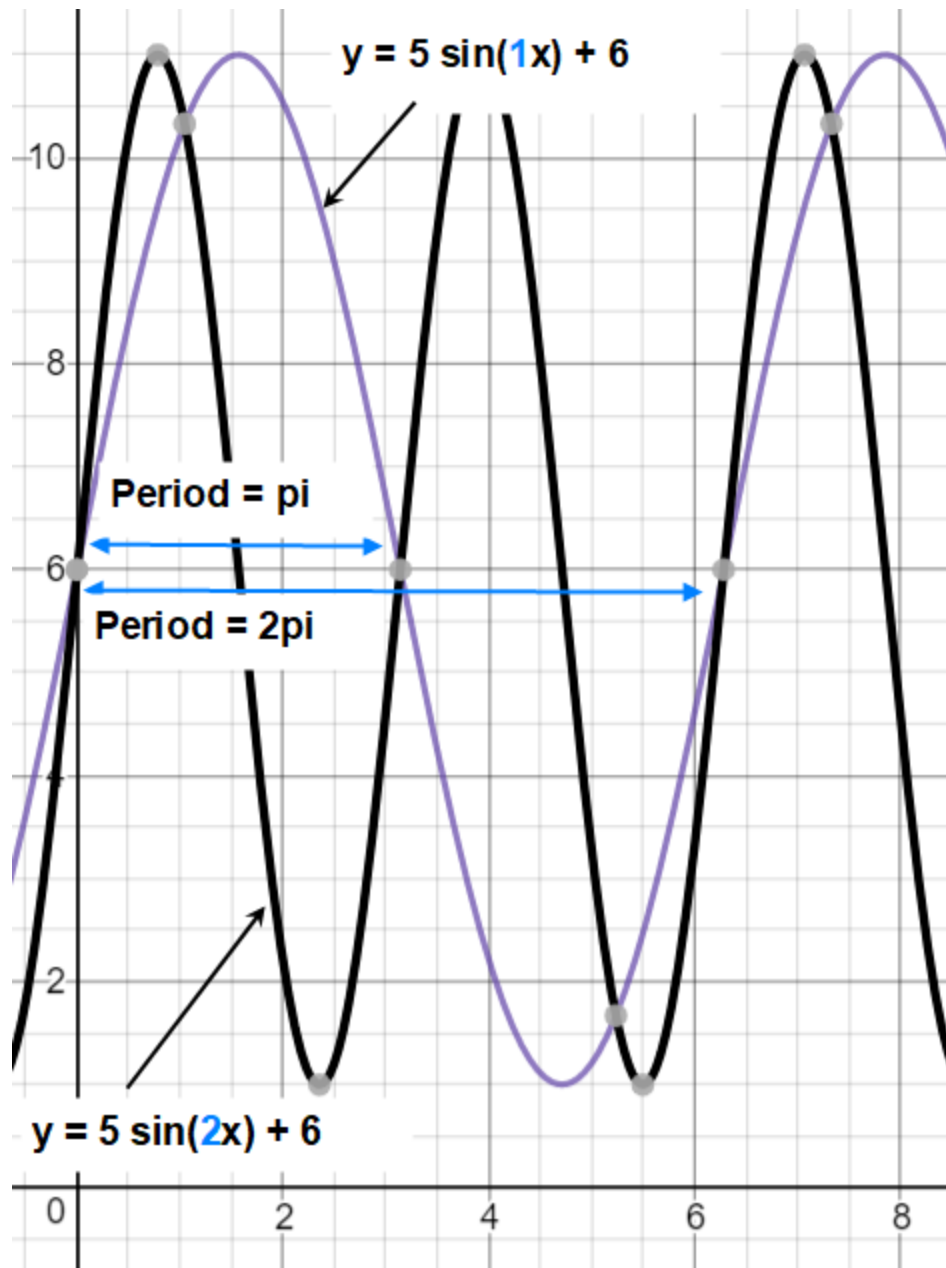


Note the midline of  $y = 5 \sin(x)$  is the x-axis ( $y=0$ ). The vertical shift is up 6 units so the new midline is  $y = 6$ . The period is still the same ( $2\pi$ ).

## Graphing ... one step at a time ... applying the new period

**Step 4:** Change the period resulting in:

$$y = 5 \sin(2x) + 6$$



The period calculation:

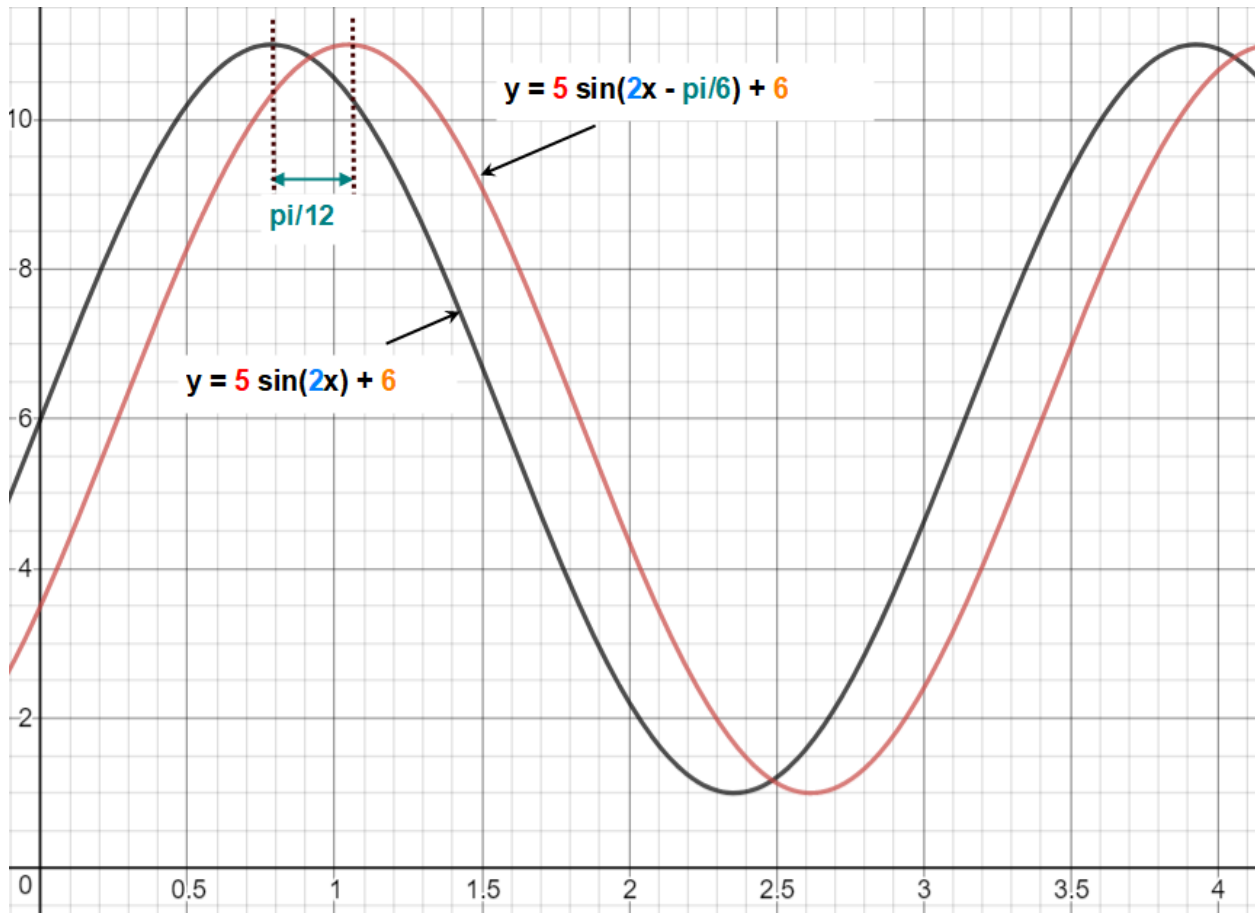
$$\text{Period} = 2\pi/B \quad (\text{where } B \text{ is } 2 \text{ in this example}) = 2\pi/2 = \pi$$

Above the purple function has the new period of  $\pi$ , which is half the period of the black function.

## Graphing ... one step at a time ... applying the new phase shift

**Step 5:** Change the phase shift resulting in:

$$Y = 5 \sin(2x - \pi/6) + 6$$



Note that above the black function has shifted to the right by  $\pi/12$  resulting in the red function.

Phase shift calculation:

$$\text{Shift} = C/B \text{ (where } C \text{ is } \pi/6 \text{ and } B \text{ is } 2) = 2/(\pi/6) = \pi/12$$

## Conclusion ... the completed graph

All the transformations have been applied: Amplitude, Vertical Shift, Period and Phase Shift:

$$Y = 5 \sin(2x - \pi/6) + 6$$

