Derivation of quadratic formula & x coordinate of parabola's vertex

$$ax^2 + bx + c = 0$$

Solve for x (derive the quadratic formula) . . .

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

complete the square . . .

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

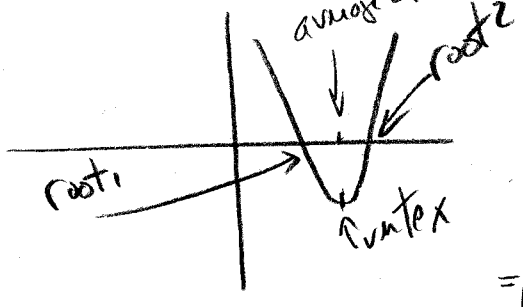
factor $$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

take sq root $$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a} = \boxed{\frac{-b \pm \sqrt{b^2-4ac}}{2a}}$$

We now have a general expression for the roots of the quadratic (parabola). The vertex of the parabola is exactly in the middle of these two roots (horizontally). The x coordinate of the vertex then is the average value of the roots.



Calculate the midpt (n average value) of roots. This will be x coordinate of the vertex:

$$= \left[\frac{-b - \sqrt{b^2-4ac}}{2a} + \frac{-b + \sqrt{b^2-4ac}}{2a}\right]\frac{1}{2}$$

$$= \left(\frac{-2b}{2a}\right)\left(\frac{1}{2}\right) = \boxed{-\frac{b}{2a}} \leftarrow x \text{ coordinate of parabola}$$